

# Two-dimensional solitons with second-order nonlinearities

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We investigate the propagation of intense Gaussian beams in materials with quadratic nonlinearity. Excitation of  $(2 + 1)$  solitons is numerically predicted at finite phase mismatch and in the presence of linear walk-off between the fundamental and second-harmonic waves. The numerical results are interpreted in terms of the conserved quantities of the wave evolution, and the appropriate conditions for the experimental observation of the solitons are discussed.

In addition to harmonic generation and frequency mixing, intense light beams propagating in  $\chi^{(2)}$  nonlinear media exhibit a variety of self- and cross-phase modulation effects that until recently were commonly thought to occur only in  $\chi^{(3)}$  interactions. One important example is the propagation of soliton-like waves, in which the fundamental and second-harmonic waves mutually focus and trap each other. The  $(1 + 1)$  trapping (i.e., one transverse dimension and one propagation dimension) yields both spatial and temporal solitons that can in principle occur in planar waveguides and fibers, respectively.<sup>1,2</sup> Although spatial solitons appear to be easier than temporal solitons to obtain experimentally,<sup>3</sup> their excitation still requires nearly phase-matched propagation in high-quality planar waveguides. Here we study the propagation of cw Gaussian beams in bulk  $\chi^{(2)}$  media, and we show that it is stable under a variety of physically realistic conditions. Thus  $(2 + 1)$  confinement in  $\chi^{(2)}$  media has potential applications to switching in bulk optics and to wave propagation in cavities containing  $\chi^{(2)}$  crystals.

The mutual focusing of Gaussian beams in  $\chi^{(2)}$  media and the implications for the formation of solitons were first investigated by Karamzin and Sukhorukov.<sup>4</sup> The formation of stationary  $(3 + 1)$  states was later analyzed by Kanashov and Rubenchik.<sup>5</sup> The main conclusion of those works is that the interaction of parametric waves in  $\chi^{(2)}$  media can lead to the formation of higher-dimensional solitons (or, more properly, solitonlike waves). More recently, the problem has been revisited by Hayata and Koshiba,<sup>6</sup> who analyzed multidimensional confinement, using an approximate solitary-wave solution in a phase-matched configuration. Our goal here is to identify the conditions required for the actual excitation of  $(2 + 1)$  solitons and to show that indeed these solitons form under a variety of currently accessible experimental conditions, in the presence of finite mismatch and linear walk-off between the fundamental and second-harmonic waves.

We consider cw light beams traveling in a  $\chi^{(2)}$  active bulk crystal. We write the electric field of each of the waves in the form  $E(\mathbf{r}, t) = A(\mathbf{r})\exp(ikz - i\omega t)$ , and we make the slowly varying envelope approxima-

tion. Under such conditions, the wave propagation can be described by the equations<sup>3</sup>

$$\begin{aligned} i \frac{\partial a_1}{\partial \xi} + \frac{1}{2} \nabla_{\perp}^2 a_1 + a_1^* a_2 \exp(-i\beta\xi) &= 0, \\ i \frac{\partial a_2}{\partial \xi} + \frac{\alpha}{2} \nabla_{\perp}^2 a_2 - i\delta \frac{\partial a_2}{\partial x} + a_1^2 \exp(i\beta\xi) &= 0, \end{aligned} \quad (1)$$

where  $a_1$  and  $a_2$  are the normalized amplitudes of the fundamental and second-harmonic waves, respectively,  $\alpha = n_1/2n_2$ ,  $\delta = k_1\eta\rho$ , and  $\beta = k_1\eta^2\Delta k$ . Here  $\Delta k = 2k_1 - k_2$  is the phase mismatch,  $n_1$  and  $n_2$  are the appropriate refractive indices at both frequencies,  $\rho$  is the walk-off angle, and  $\eta$  is the beam width. The transverse coordinates are measured in units of  $\eta$ , and  $z/l_c = |\beta|\xi/\pi$ , where  $l_c = \pi/|\Delta k|$  is the coherence length. For  $l_c \sim 2.5$  mm,  $\lambda_1 \sim 1$   $\mu$ m,  $n_1 \sim 1.7$ ,  $\rho \sim 1^\circ$ , and  $\eta \sim 15$   $\mu$ m, which are typical values for current materials and lasers, one has  $\alpha \approx 0.5$ ,  $\beta \sim \pm 3$ , and  $\delta \sim 1.5$ . In this Letter we focus on such near-phase-matching conditions, reserving the large phase-mismatch regime for a later publication.

From a physical standpoint, the stability of the  $\chi^{(2)}$  higher-dimensional confinement as opposed to its  $\chi^{(3)}$  counterpart, which in Kerr  $\chi^{(3)}$  media in the paraxial approximation is always unstable, is not hard to understand. The rate at which a beam tends to focus is proportional to  $I_m$  in a Kerr  $\chi^{(3)}$  medium and to  $I_m^{1/2}$  in a  $\chi^{(2)}$  medium, with  $I_m$  being the maximum light intensity; in both cases, the rate at which a beam tends to diffract is proportional to  $\eta^{-2}$ . Because energy is conserved,  $I_m\eta^2$  may be assumed to be roughly constant when a beam with radial symmetry collapses; hence, under this assumption, the rate at which a beam diffracts is proportional to  $I_m$ . In a  $\chi^{(3)}$  medium the rate of diffraction matches the rate of self-focusing. In contrast, in a  $\chi^{(2)}$  medium the diffraction rate will always become greater than the rate at which the beam focuses, so that Eqs. (1) will not exhibit collapse. An important point that warrants emphasis is that the  $\chi^{(2)}$  nonlinearity acts like the  $\chi^{(3)}$  nonlinearity only when the coherence length is short compared with the diffractive scale length, so that the wave evolution in  $\chi^{(2)}$  media can be approximately described by the nonlinear Schrödinger

equation.<sup>1</sup> However, although self-focusing can occur in this limit, ultimately the difference between the  $\chi^{(2)}$  and the  $\chi^{(3)}$  nonlinearities must manifest itself when both scale lengths become comparable, so that, in particular, collapse does not occur.

To show that the stable higher-dimensional soliton-like waves described by Eqs. (1) do exist, we examine the following two conserved quantities of the beam evolution:

$$I = \int \{|a_1|^2 + |\hat{a}_2|^2\} dr_{\perp}, \quad (2)$$

$$\mathcal{H} = \frac{1}{2} \int \left\{ |\nabla_{\perp} a_1|^2 + \frac{\alpha}{2} |\nabla_{\perp} \hat{a}_2|^2 + \beta |\hat{a}_2|^2 - i \frac{\delta}{2} \left( \hat{a}_2 \frac{\partial \hat{a}_2^*}{\partial x} - \hat{a}_2^* \frac{\partial \hat{a}_2}{\partial x} \right) - (a_1^* \hat{a}_2 + a_1 \hat{a}_2^*) \right\} dr_{\perp}, \quad (3)$$

where we have defined  $\hat{a}_2 = a_2 \exp(-i\beta\xi)$ . The former is the Manley–Rowe relation; the latter is proportional to the Hamiltonian of Eqs. (1). We will show that, for fixed intensity and phase mismatch,  $\mathcal{H}$  is bounded from below. We restrict ourselves to the case  $\delta = 0$ . Using the approach developed by Zakharov and Kuznetsov<sup>7</sup> (see also Refs. 5 and 8), one finds that

$$\mathcal{H} \geq -\frac{1}{4} (|\beta| - \beta) I - I^2.$$

This is a crude estimate. Nevertheless, as the stationary solutions of Eqs. (1) can be obtained as the extrema of the quantity  $\{\mathcal{H} + \kappa_1 I\}$ , with  $\kappa_1$  being the nonlinear correction of  $k_1$  that is due to the wave interaction, this bound indicates that a stable solution exists that realizes the absolute minimum. In fact, at exact phase matching one can readily show that the stationary solutions are realized at  $\mathcal{H} = -\kappa_1 I/2 < 0$ .<sup>5</sup> Therefore, as expected from physical grounds, a second-harmonic signal has to be supplied at the input face of the crystal for such stationary solitons to be excited because otherwise  $\mathcal{H} > 0$ . A family of stationary solutions could in principle be found by solution of the stationary governing equations. However, the stationary solutions do not necessarily exhaust the possible (2 + 1) solitonlike wave behavior generated by the beam interaction. Indeed, oscillating solitonlike waves, with the energy going back and forth between the interacting waves, could exist, as they do in (1 + 1) trapping.<sup>2</sup>

In order to demonstrate that stable beam propagation occurs for a wide range of practically realizable conditions, we numerically solved Eqs. (1), using a split-step Fourier approach. In all our simulations we used Gaussian input beams:

$$a_1(\xi = 0) = A \exp(-r_{\perp}^2),$$

$$a_2(\xi = 0) = B \exp(-r_{\perp}^2).$$

We begin by analyzing the beam evolution with negligible walk-off ( $\delta = 0$ ). We set  $\alpha = 0.5$ ,  $\beta = \pm 3$ , and we monitor the beam evolution at various input intensities. Figure 1 shows the outcome for the

fundamental wave. Below a threshold input intensity, the beams spread. In contrast to this, when suitable signals are injected with sufficient intensity at the input face of the crystal the beams evolve into a solitonlike wave. Solitons form with both signs of the phase mismatch, but for identical excitation conditions the sign of  $\beta$  significantly affects the beam evolution. In particular, in our numerical experiments at  $B = 0$  with negative phase mismatch the beams always spread. For comparison, we have also plotted the evolution of a beam governed by the nonlinear Schrödinger equation (NLS). Above the collapse threshold, formation of the characteristic spike is clearly visible. In a second numerical experiment we examine the beam evolution in the presence of moderate linear walk-off. We set  $\delta \sim 1$ , which corresponds to a walk-off length comparable with the diffractive scale length. As is the case for (1 + 1) trapping,<sup>2</sup> we find that the beams stick together and evolve into a soliton. Figure 2 is representative of the beam evolution in that regime.

Under the conditions in which Eqs. (1) are applicable, the experimental observation of mutual

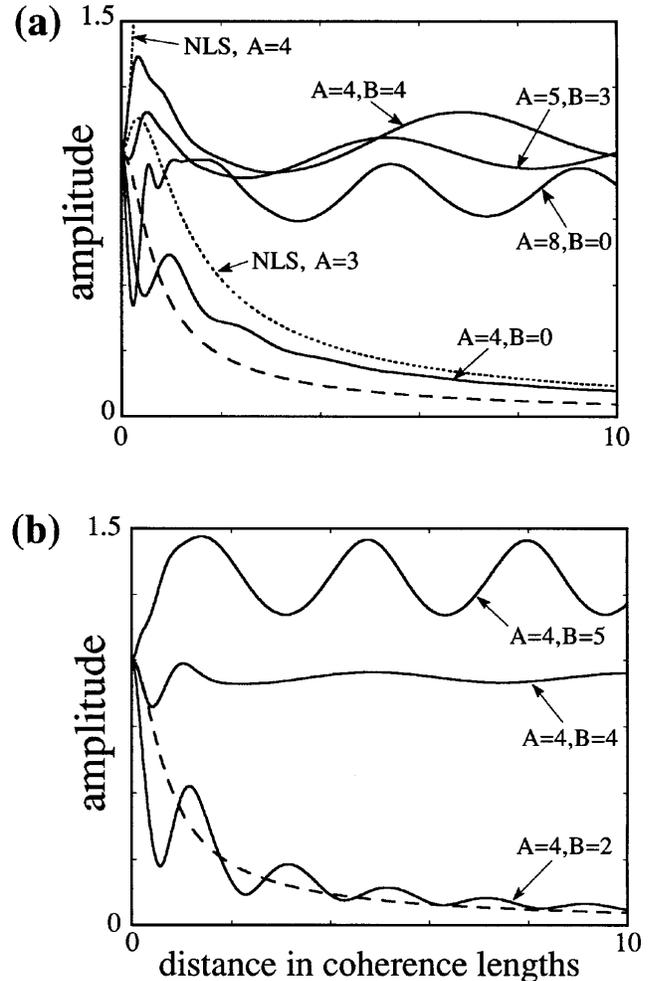


Fig. 1. Evolution of the amplitude of the fundamental wave. Dashed curves show the evolution of the beams in absence of nonlinearity. Dotted curves show the evolution of a beam governed by the nonlinear Schrödinger equation. All values are scaled to the input amplitudes. (a)  $\beta = 3$ , (b)  $\beta = -3$ .

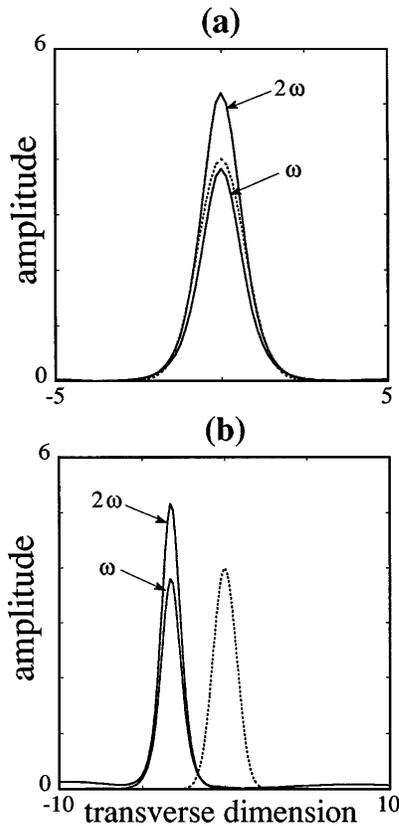


Fig. 2. Detail of the beam evolution in the presence of walk-off. The plots show a slice of the beams along the walk-off axis. Dashed curves,  $\xi = 0$ ; solid curves,  $\xi = 5$ . (a)  $\delta = 0$ , (b)  $\delta = 1$ . In both cases  $\beta = -3$ ,  $A = 4$ , and  $B = 4$ .

beam trapping requires two primary ingredients: large, nonresonant  $\chi^{(2)}$  nonlinearities, which have to be accessible in configurations with negligible or moderate walk-off, and a large damage threshold and long samples. As suggested in Ref. 6, such conditions can be fulfilled by ferroelectric materials such as lithium niobate and potassium titanyl phosphate, although photorefractive effects could limit the usefulness of lithium niobate. For the parameter values in Figs. 1 and 2 the phase-matchable nonlinear coefficients of these materials accessible through birefringence tuning lead to peak intensities in the range 0.1–1 GW/cm<sup>2</sup>, which are accessible with common picosecond pulsed lasers. However, organic materials, with larger nonlinear coefficients, constitute a promising alternative. The large walk-off characteristic of these materials limits their usefulness for the present purposes to spectral ranges in which noncritical phase matching occurs. Noncritical configurations have been identified, for instance, in *N*-4-nitrophenyl-*L*-prolinol,<sup>9</sup> with effective coefficients as large as 70 pm/V. For such nonlinear coefficients one gets peak intensities of the order of 1 MW/cm<sup>2</sup>, which are readily reached in tightly focused geometries with cw lasers.

In conclusion, we have investigated the formation of (2 + 1) solitons with intense cw beams in materials with second-order nonlinearity. We have

explored here near-phase-matching conditions. We have observed numerically that (2 + 1) solitonlike waves form at both signs of the phase mismatch and continue to exist in the presence of moderate linear walk-off between the fundamental and the second-harmonic waves. At the large-phase-mismatch regime, at which under certain conditions the wave evolution can be approximately described by the nonlinear Schrödinger equations, the beams do not collapse. The detailed beam evolution in that regime remains to be analyzed. As we have assumed input cw lasers, we have ignored here the effects on pulsed signals that are due to chromatic dispersion and different group velocities at both frequencies. However, we recently found that, when the two transverse spatial coordinates can be ignored, temporal solitonlike waves form also in the presence of moderate temporal walk-off.<sup>2</sup> The combination of both effects to yield stable two-wave light bullets through the modulational instability of the (2 + 1) bound states is an interesting possibility<sup>5</sup> that we will investigate.

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