Spectral-Spatial Hyperspectral Unmixing Using Nonnegative Matrix Factorization

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Abstract—Remotely sensed hyperspectral images contain several bands (at about adjoining frequencies) for a similar zone on the surface of the Earth. Hyperspectral unmixing is a significant method for breaking down hyperspectral images into the components (endmembers) that conform each (potentially mixed) pixel and their abundance maps. Nonnegative matrix factorization (NMF) has attracted huge consideration because of the way that it can address mixed pixel scenarios. Most existing NMF unmixing techniques do not include spatial information in the analysis. An ongoing trend is to fuse the spatial and the spectral information contained in hyperspectral scenes to improve the solution. In this paper, we build up another hyperspectral unmixing technique named spatial-spectral weighted nonnegative matrix factorization (SSWNMF), in which two weighting factors are acquainted into the NMF model to upgrade the sparsity of the solution and capture the piecewise smooth structure of the data. We adopt a multiplicative iterative strategy to implement the proposed SSWNMF model. Our experimental results, conducted with both synthetic and real hyperspectral data, uncover that the proposed SSWNMF strategy can get accurate unmixing results over those gave by other unmixing strategies, with less parameter tuning.

Index Terms—Hyperspectral unmixing, blind source separation (BSS), nonnegative matrix factorization (NMF), weighted sparsity, spatially-weighted unmixing.

I. INTRODUCTION

Hyperspectral sensors can detect signals with fine spectral resolution, hence advancing the improvement of quantitative far off detecting applications. Hyperspectral imaging has been broadly applied in military and civilian fields [1]. Due to the (frequently restricted) spatial resolution of imaging spectrometers, mixed pixels are abundant in hyperspectral images [2]. To manage this issue, hyperspectral unmixing has been broadly used to recognize the pure components in a scene (endmembers) and to appraise the abundance maps of such unadulterated signatures in each blended pixel [3].

In the previous decades, numerous linear mixture model (LMM)-based unmixing techniques have been created, which incorporate two separate categories: endmember extraction and abundance estimation. These calculations can be partitioned into two primary gatherings. From one perspective, some endmember extraction strategies depend on the presumption that the scene contains one unadulterated (pure) pixel, for example, vertex component analysis (VCA) [4], pixel purity index (PPI) [5], N-FINDR [6], simplex growing calculation (SGA) [7], among numerous others. Actually, because of the multifaceted nature of ground objects, unadulterated pixels scarcely appear in hyperspectral scenes. To address this issue, other endmember extraction techniques dropped this presumption by expecting that unadulterated pixels may not exist in the hyperspectral scenes. For example, simplex growing approaches [8] include: the minimum volume enclosing simplex (MVES) [9], the minimum volume simplex analysis (MVSA) [10], and the simplex identification via split augmented Lagrangian (SISAL) [11]. These calculations will in general get virtual endmembers without physical relevance. The exactness of the endmember extraction process will affect the abundance inversion results.

From the perspective of signal processing, hyperspectral unmixing can be viewed as a blind source separation (BSS) problem [12]. For the issue of BSS, nonnegative matrix factorization (NMF) is a regular arrangement, and it has as of late been utilized for hyperspectral unmixing [13]. NMF-based unmixing targets decomposing hyperspectral images into two parts at the same time, one of which is the matrix of endmembers and the other is the abundance matrix [14]. In addition, because of its characteristics, NMF is especially reasonable for the scene unmixing task [15].

In this paper, we focus on hyperspectral unmixing using NMF. The objective function of standard NMF is non-convex and with scale uncertainty. Thus, if the calculation just considers the non-negativity constraint, it might fall into a local minimum. To improve the performance of the NMF, an assortment of extra constraints are considered in the standard NMF model. Among them, one line is to force constraints on the endmembers, while other strategies will force constraints on the abundances. A combination of both actions is additionally utilized in certain developments.

For the NMF-based unmixing strategies that force constraints on the endmembers, a procedure is to investigate the inherent structure of the whole endmember set. For instance, the minimum volume constraint (MVC) was incorporated into the NMF model, prompting the MVC-NMF hyperspectral unmixing technique [16]. Wang et al. proposed an endmember dissimilarity constrained NMF technique (EDC-NMF) [17],
which utilizes the endmember uniqueness capacity to locate smooth spectra from the dataset with the biggest distinction so as to accomplish hyperspectral unmixing. Huck et al. proposed a minimum dispersion-constrained NMF (MiniDisCo) technique [18] to unmix hyperspectral images, which addresses the unmixing issue utilizing endmember spectra ought to have least fluctuations.

For the NMF-based unmixing strategies that force constraints on the abundances, since most pixels in hyperspectral images are mixed by a subset of endmembers, the abundance matrix exhibits sparsity [19]. Under this presumption, a sparse constraint was acquainted into the NMF model. From a scientific point of view, the \( L_0 \) regularizer is a direct strategy that delivers the sparsest outcomes. In any case, the \( L_0 \) regularizer is non-convex and hard to compute, and it is typically replaced by the \( L_1 \) regularizer. At that point, the \( L_1 \) regularizer is often replaced by the \( L_p \) (for \( 0 \leq p < 1 \)) regularizer [20] to accomplish greater sparsity results. Xu et al. outlined that \( L_{1/2} \) was productive and could be taken as a surrogate of the \( L_p \) (\( 0 < p < 1 \)) regularizer [21]. Enlivened by this, the \( L_{1/2} \) regularizer was brought into the NMF model for unmixing, in order to confine the sparsity of abundance maps [22]. In [23], the graph-regularized \( L_{1/2} \)-NMF (GLNMF) technique was proposed to distinguish the complex structure of the hyperspectral information. Since this technique consolidates sparsity limitations into the NMF model, it gives preferred outcomes over other inadequate NMF calculations. In [24], the robust collaborative NMF (R-CoNMF) acquainted the \( L_{2,1} \) regularizer to promote row-sparsity. To additionally improve the sparsity of abundance maps, a weighting methodology was embraced to reduce the nonzero coefficients on the sparse solution. In [25], a weighted \( L_1 \) regularizer was added to the NMF model, and it displays better outcomes than the non-weighted \( L_1 \) regularizer. Besides, a double weighted sparse regularizer was proposed, in which two weights are utilized to improve the column-sparsity and row-sparsity of the abundance matrix, individually [26].

Truth be told, hyperspectral images hold rich spatial information. A few examinations have demonstrated that fusing spatial data into the NMF-based unmixing model can all the more likely exploit endmember properties [27]. Following this perception, a few calculations have concentrated on improving unmixing execution by exploiting the spatial-relevant data contained in the hyperspectral data cube. In [28], abundance separation and smoothness constraints are brought into the NMF (ASSNMF) model for hyperspectral unmixing. In [29], local neighborhood weights are acquainted into the NMF model. In [30], structured sparse-regularized NMF (SSNMF) technique was proposed, which consolidates a graph Laplacian to encode the complex structures implanted in the hyperspectral manifold space to learn an exact arrangement of the endmembers. In [25], the total variation (TV) regularized reweighed sparse NMF (TV-RSNMF) technique has been proposed, which utilizes the TV regularizer to catch the piecewise structure of the abundance matrix. Additionally, in the field of sparse unmixing (as a semi-supervised method, it relies on a previously available spectral library), the spatial weighting factor is combined with the sparse regularizer to portray the spatial structure of the data. For example, in [31], the local collaborative sparse unmixing (LCSU) has been proposed, which utilizes a spatial weight to obtain collaborativity and accomplish great unmixing performance. Besides, in [32], the spectral-spatial weighted sparse unmixing (S^3WSU) strategy has been proposed, which utilizes spatial weighting components to exploit the spectral and spatial data contained in hyperspectral images.

Enlivened by LCSU and S^3WSU calculations, this paper introduces a new blind hyperspectral unmixing method named spectral-spatial weighted sparse NMF (SSWNMF). The proposed SSWNMF uses both spatial and spectral information and the sparse structure of hyperspectral images, which is achieved in both the spectral and the spatial domain under the \( L_1 \)-NMF formulation. The main contributions of our work can be summarized as follows:

1) Our new SSWNMF enhances the sparsity and spatial smoothness of abundance maps by introducing weighting factors in the \( L_1 \)-NMF unmixing model. On the one hand, because of the way in which a subset of endmembers is normally utilized to unmix a pixel (rather than the full set of endmembers), the abundance matrix exhibits row-sparsity. Subsequently, we introduce a spectral weighting factor to upgrade the sparsity of nonzero lines of the abundance matrix in the NMF-based unmixing model. Then again, as referenced above, hyperspectral images contain rich spatial information, and the pixels inside a neighborhood spatial gathering are relied upon to impose spatial constraints in the abundance maps. At that point, the spatial weighting factor consolidates the spatial connections to characterize piecewise-smooth changes in the abundance maps, following a local strategy. These are both innovative contributions with regards to previous works. In addition, our newly proposed SSWNMF can provide sparser representations than other sparsity-inducing regularizers (for example, the \( L_{1/2} \)-standard and the \( L_{1/2} \)-standard) with the inclusion of spectral and spatial weighting factors.

2) From the viewpoint of blind unmixing, an important innovative aspect of the proposed SSWNMF lies in its open structure, which can accept a variety of spectral and spatial weighting factors under the \( L_1 \)-NMF framework, thus providing great flexibility in the tasks of enhancing the sparsity of the abundance matrix and exploring various spatial scenarios (such as nonlocal similarity, superpixel homogeneous regions, morphological structural information, etc.). These are also significant innovations with regards to previous related contributions.

3) Concerning its computational multifaceted nature, the proposed SSWNMF can be effectively resolved by a multiplicative iterative strategy [33], with comparable complexity to that of the \( L_1 \)-NMF unmixing technique. In addition, since there is only one regularizer term in our SSWNMF model, it is more efficient than other NMF-based unmixing approaches such as GLNMF [23], TV-RSNMF [25] and ASSNMF [28]. Because of the
incorporation of the spatial information and the sparse regularizer, SSWNMF can get better unmixing results than other NMF-based calculations. This shows that the proposed SSWNMF method is simple but very effective, as opposed to other related methods that exhibit highest complexity. This is a third innovative aspect of our new contribution.

The rest of this paper is composed as follows. Section II gives a concise presentation of the LMM and the NMF calculations. The proposed SSWNMF unmixing technique (and its solution algorithm) are introduced in Section III. Sections IV and V portray the experimental results with synthetic and real hyperspectral data, and give thorough assessments. At long last, section VI finishes up this paper with certain comments and alludes to conceivable future examination lines.

II. BACKGROUND

In this section, the linear mixing model (LMM) will be quickly presented. Then, the standard NMF model and the sparsity regularizer-based NMF model will be examined.

A. Linear Mixing Model (LMM)

Our proposed SSWNMF unmixing method depends on the LMM. Here, we give insights regarding the LMM. The LMM assumes that the spectral composition of a pixel is shaped by a direct mix of the unadulterated materials (endmembers) present in the pixel. Let a hyperspectral image be indicated as a direct mix of the unadulterated materials (endmembers) where the spectral composition of a pixel is shaped by a direct mix of the unadulterated materials (endmembers) present in the pixel. Let a hyperspectral image be indicated as $Y = [y_1, \ldots, y_N] \in \mathbb{R}^{d \times N}$, where $d$ is the number of bands, $N$ signifies the number of pixels in the picture, and $y_i$ is a $d$-dimensional vector which represents the $i$th pixel. The LMM can be expressed by:

$$y = Ms + n,$$  

where $M \in \mathbb{R}^{d \times m}$ is the endmember matrix and $s = [s_1, \ldots, s_m]^T$ means the abundance vector of the pixel, the operation $(\cdot)^T$ denoting matrix transpose, $m$ is the number of endmembers, and $n$ is the noise. The unmixing problem needs to fulfill two physical imperatives: the abundance non-negativity (ANC) (i.e., $s_i \geq 0$), and the abundance sum-to-one (ASC) (i.e., $\sum_{i=1}^{m} s_i = 1$) constraints.

Adopting matrix notation, the hyperspectral data set $Y$ can be described in matrix form as:

$$Y = MS + N$$  

where $S = [s_1, \ldots, s_N] \in \mathbb{R}^{m \times N}$ indicates the abundance matrix, and $N = [n_1, \ldots, n_N] \in \mathbb{R}^{d \times N}$ is the noise matrix. Strikingly, under the LMM, since just the hyperspectral image $Y$ is given, the decomposition of $Y$ is a BSS problem, whose object is to recoup the endmember signature matrix and the associated abundance matrix.

B. Nonnegative Matrix Factorization (NMF)

As NMF has numerous points of interest, it has gotten impressive consideration in the field of hyperspectral unmixing. The reason for NMF is to roughly break down a large nonnegative matrix $Y$ into two nonnegative matrices $M$ and $S$ [34]. The objective function of the NMF strategy is as per the following:

$$\min_{M,S} \frac{1}{2} ||Y - MS||_F^2 \quad \text{s.t.} \quad M \geq 0, \quad S \geq 0, \quad 1_m^T S = 1_N^T,$$  

where $|| \cdot ||_F$ is the Frobenius norm, and $1_m^T$ and $1_N^T$ indicate all one vectors with size $m$ and size $N$, individually. For the solution of (3), in spite of the fact that it is non-convex, its two subproblems related to $M$ and $S$ are convex, which can for the most part be solved by alternatively updating $M$ and $S$.

Gradient descent is a notable technique that can take care of the optimization problem (3). In any case, it is very hard to decide the size of each step [35]. In [33], Lee and Seung proposed a multiplicative iterative way to take care of this issue, and gave a convergence proof. The update rules are as per the following:

$$M \leftarrow M * Y S^T / MSS^T,$$  

$$S \leftarrow S * M^T Y / M^T MS,$$  

where $*$ and $/$ are the element wise multiplication and division, separately. Note that, when the underlying $M$ and $S$ are nonnegative, the non-negativity of the two matrices will continue as before under the guidelines given by (4) and (5). However, because of the way that the objective function in (3) is not convex with regards to $M$ and $S$, it will prompt the presence of numerous local minima. Consequently, while applying NMF to hyperspectral unmixing, more limitations should be thought of.

C. NMF with a Sparse Regularizer

Since a mixed pixel for the most part results from the mix of just a few endmembers, the abundance matrix shows sparsity [22]. At the end of the day, abundance sparsity is an inherent property of hyperspectral data, and it has been investigated in numerous NMF-based hyperspectral unmixing approaches. To put it plainly, the sparsity is a constraint that advances the uniqueness of the decomposition and assumes a significant job in improving abundance estimation. The corresponding objective function is characterized as follows:

$$\min_{M,S} \frac{1}{2} ||Y - MS||_F^2 + \lambda h(S),$$  

s.t. $M \geq 0, \quad S \geq 0, \quad 1_m^T S = 1_N^T,$

where $\lambda \geq 0$, $S \geq 0$, $1_m^T S = 1_N^T$, and $h(\cdot)$ denotes the sparsity regularizers following up on the abundance matrix $S$. Thinking about the sparsity of the abundance matrix, the sparsity requirement is brought into the standard NMF model. The $L_1$ regularizer is generally used to play out NMF-based hyperspectral unmixing. Be that as it may, when the full additivity constraint is forced on the abundance matrix, the $L_1$ norm can’t drive further sparsity. So as to all the more likely describe the sparsity of the abundance matrix, numerous
old style sparse NMF unmixing techniques have been created. For instance, the $L_{1/2}$-NMF [22] and the R-CoNMF [24] calculations center around presenting new orders on the sparse regularizer to improve the sparsity of the abundance fractions. In addition, because of the spatial course of action of pixels in an image, the corresponding abundance shows articulated spatial autocorrelation. Some advanced spatial-based sparse NMF unmixing techniques have been proposed, for example, the TV-RSNMF [25] and the spatial group sparsity regularized NMF (SGSNMF) [13], whose design is to model piece-wise transitions in the estimated abundances.

III. PROPOSED SPECTRAL-SPATIAL WEIGHTED SPARSE NMF METHOD

As referenced over, the two earlier suppositions of spatial relationship and sparsity assume a significant job in improving the performance of hyperspectral unmixing. In this section, the sparse prior and spatial information are coordinated into the NMF model to compel the solution space of the abundance matrix, and our new SSNMF unmixing model is proposed.

A. The Spectral-Spatial Weighted Sparse NMF model

Motivated by the accomplishment of weighted $L_1$ minimization in improving signal sparsity [36], and furthermore by the achievement of the double reweighted sparse unmixing with total variation (DRSU-TV) [26], in this work we fuse the spectral and spatial weighting factors into the $L_1$-NMF model, and the objective function is characterized as follows:

$$\min_{M,S} \frac{1}{2} ||Y-MS||_F^2 + \lambda ||(H_{spe}H_{spa}) \odot S||_{1,1},$$

s.t. $M \geq 0, S \geq 0, 1_M^T S = 1_N^T$,

where $||S||_{1,1} = \sum_{j=1}^{N} ||s_j||_1$ with $s_j$ being the $j$-th column of endmember abundance matrix $S$. The operator $\odot$ is the element-wise multiplication of two factors, and $\lambda \geq 0$ means the regularization parameter. Let $H_{spe} \in \mathbb{R}^{m \times m}$ be the spectral weighting factor, which expects to advance the sparsity of the rows in the abundance matrix, as demonstrated as follows:

$$H_{spe}^{t+1} = \text{diag}\left[\frac{1}{||S'(1,:)||_2 + \varepsilon}, \ldots, \frac{1}{||S'(m,:)||_2 + \varepsilon}\right],$$

where $S'(i,:)$ means the $i$-th row in the evaluated matrix at the $t$-th iteration, and $\varepsilon > 0$. Note that, as appeared in Eq. (8), it is recommended that huge estimations of $H_{spe}$ disharpen nonzero entries in the assessed abundance matrix, while small weights can be utilized to support nonzero entries [26].

Besides, taking into account the significance of considering spatial data for NMF unmixing, we consider the way that hyperspectral images ordinarily show high spatial correlation among adjoining pixels, which implies that the abundance map can be viewed as piecewise smooth. With this in mind, similar to the $S^2$WSU calculation in [32], for the spatial weighting factor $H_{spa} \in \mathbb{R}^{m \times N}$ we let $h_{spa,ij}$ be the component of the $i$-th line and $j$-th row in $H_{spa}$ at iteration $t + 1$. We consider the spatial neighbors as follows:

$$h_{spa,ij}^{t+1} = \frac{1}{f_N(s_{ij})(s_{ij}^T) + \varepsilon},$$

where $N(s_{ij})$ represents the adjacent set for element $s_{ij}$ ($i = 1, \ldots, m, j = 1, \ldots, N$). In other words, it represents the neighborhood of the $j$-th pixel (i.e. $s_{ij}$) in the $i$-th row of the abundance matrix. $f(\cdot)$ is a function that explicitly exploits the spatial correlations through the neighborhood system. Here, both neighbor coverage and importance are used to incorporate the spatial correlation as follows:

$$f_N(s_{ij})(s_{ij}) = \frac{\sum_{h_{ij} \in N(s_{ij})} \theta_h \theta_b}{\sum_{h_{ij} \in N(s_{ij})} \theta_h},$$

where $\theta$ models to the neighboring significance. Here, an eight-connected $(3 \times 3$ window) is utilized for the analysis. Note that, for the center pixel $s_{ij}$ and its neighboring pixels $s_b$, we have $\theta_b = \frac{1}{\prod_{i,j \in N(s_{ij})}}$, where the function $\prod_{i,j}(\cdot)$ indicates the significance of the two components $s_b$ and $s_{ij}$. Let $(e,g)$ and $(k,m)$ be the spatial coordinates of $s_b$ and $s_{ij}$. We consider the Euclidean distance, that is, $\theta_b = 1/\sqrt{(e-k)^2 + (g-m)^2}.$

When compared to other sparse NMF hyperspectral unmixing techniques, there are two significant contributions introduced by our SSNMF. The first one is to bring the spectral weighting factor into the sparse NMF model, which is more viable than the $L_1$ (or, potentially $L_2,1$) regularizers in the task of upgrading the row sparsity of the abundance matrix. Another important contribution is the introduction of the spatial weighting factor into the NMF model, which incorporates spatial relationships through a weighting factor (instead of utilizing the spatial prior regularization technique, as in TV-based sparse NMF hyperspectral unmixing strategies), bringing about less regularization parameters and lower computational complexity.

B. Solution of the Optimization Problem

The optimization (7) is like the NMF case, which is not convex as for $M$ and $S$ together. Hence, it is exceptionally hard to locate the global maximum. To address this issue, the optimization of (7) is part into two convex subproblems and the procedure is solved by a multiplicative iterative calculation [33]. The two split subproblems are given in Eqs. (11) and (12), separately.

$$M = \arg \min_M J(M,S)$$

$$S = \arg \min_S J(M,S)$$

The proposed strategy incorporates two stages: endmember estimation and abundance estimation. In each progression, one variable is restrictively updated by the current estimation of different factors, along these lines decreasing the estimation of the objective function iteratively. The point by point portrayal is as per the following.

1) Endmember estimation step. The objective minimization function is as follows:
\[ J(M) = \arg \min_M \frac{1}{2}\|Y - MS\|^2_F + \text{Tr}(\Psi M), \quad (13) \]

where \( \text{Tr}(\cdot) \) means the trace of a matrix. It ought to be noticed that the nonnegative constraints are added to the objective function (13). Let \( \Psi \in \mathbb{R}^{d \times m} \) be the Lagrange multiplier in matrix format. At that point, we take the derivative with respect to \( M \) as follows:

\[ \nabla_M J(M) = \text{MSS}^T - YS^T + \Psi. \quad (14) \]

Based on the Karush-Kuhn-Tucker condition, it follows that:

\[ (\text{MSS}^T - YS^T) \ast M = 0. \quad (15) \]

By applying transposition and division, the update rule of endmember estimation can be obtained as follows:

\[ M \leftarrow M \ast (YS^T)/(\text{MSS}^T). \quad (16) \]

2) Abundance estimation step. The objective minimization function is formulated as:

\[ J(S) = \arg \min_S \frac{1}{2}\|Y_f - M_f S\|^2_F + \lambda \|(H_{spe} H_{spa}) \odot S\|_{1,1} + \text{Tr}(\Phi S), \quad (17) \]

where \( Y_f \) and \( M_f \) represent the augmented matrices: \( Y_f = [Y \delta 1_N^T], M_f = [M \delta 1_M^T] \). \( \Phi \in \mathbb{R}^{m \times N} \) denotes the Lagrange multiplier in matrix format, and the spectral weighted matrix \( H_{spe} \) and the spatial weighted matrix \( H_{spa} \) are estimated as shown in (8) and (9), respectively. Then, taking the derivative with respect to \( S \), we have:

\[ \nabla_S J(S) = M_f^T M_f S - M_f^T Y_f + \lambda (H_{spe} H_{spa}) \ast S = 0. \quad (18) \]

Mathematically, Based on the Karush-Kuhn-Tucker condition, \( \Phi_{m,N} S_{m,N} = 0 \), it follows that:

\[ (M_f^T M_f S - M_f^T Y_f + \lambda (H_{spe} H_{spa})). \ast S = 0. \quad (19) \]

Similarly, by applying transposition and division, the update rule of \( S \) is as follows:

\[ S \leftarrow S \ast (M_f^T Y_f)/(M_f^T M_f S + \lambda (H_{spe} H_{spa})). \quad (20) \]

In summary, the SSWMNF algorithm is given in Algorithm 1.

**IV. Experiments with Synthetic Data**

In this section, we use synthetic hyperspectral data to confirm the unmixing execution of the proposed SSWMNF approach. We compared the SSWMNF presented in this work with other progressed unmixing methods, explicitly: SGSNMF (spatial group sparsity regularized NMF) [13], TV-RSNMF [25], RSNMF [25], GLNMF [23], \( L_{1/2} \)-NMF [22] and VCA-FCLS (fully constrained least-squares) [4]. Among these looked at strategies, \( L_{1/2} \)-NMF is an exemplary sparsity-constrained NMF calculation, in which an extra sparse regularizer is forced on every column of the abundance matrix. The RSNMF calculation acquaints a weighted sparse regularizer that forces sparsity on the arrangement. In light of the RSNMF model, the TV-RSNMF strategy acquaints the TV spatial regularizer to capture the piecewise smoothness of the abundances. The SGSNMF algorithm introduces the simple linear iterative clustering (SLIC) superpixel segmentation technique to generate spatial groups, thereby describing the local distribution of endmembers. In addition, the GLNMF technique installs the manifold structure in the sparsity-constrained NMF model to protect the close connection between the pixels and the endmember abundances.

Two measurements: spectral angle distance (SAD) and root mean square error (RMSE), are embraced for quantitative evaluation. SAD is determined by the cosine of the spectral angle between the evaluated endmember \( \hat{M}_i \) and the reference endmember \( M_i \); its equation is displayed in Eq. (21). RMSE is determined by the difference between the estimated abundances \( \hat{S}_i \) and the ground-truth ones \( S_i \), where \( S_i \) represents the ground-truth abundance matrix of the \( i \)th endmember and its formula appears in Eq. (22).

\[ \text{SAD}_i = \arccos \left( \frac{M_i^T \hat{M}_i}{\|M_i^T\| \|\hat{M}_i\|} \right). \quad (21) \]
\[ \text{RMSE}_i = \sqrt{\frac{1}{N} \|S_i - \hat{S}_i\|^2}. \quad (22) \]

The smaller the SAD or RMSE is, the more accurate the unmixing is.

**A. Simulated data sets**

As appeared in Fig. 1(a), nine spectral signatures were chosen from the United States Geological Survey (USGS) library 1 for our tests. The USGS library contains 224 spectral groups, and conveyed consistently in the span 0.4-2.5 \( \mu \)m. The simulated images, produced as portrayed in [37], comprised of 100 \times 100 pixels and 224 bands for each pixel. The abundances fulfill the ANC and the ASC and show piecewise smoothness, i.e., they are smooth with sharp advances. The spatial qualities of these information can verify the performance of various

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Fig. 1. Fractional values of the endmembers in the simulated data. (a) Spectral signatures used to simulate the data. Abundance map of (b) endmember 1, (c) endmember 2, (d) endmember 3, (e) endmember 4, (f) endmember 5, (g) endmember 6, (h) endmember 7, (i) endmember 8, (j) endmember 9.

TABLE I
AVERAGE SAD SCORES (ALONG WITH THEIR STANDARD DEVIATIONS) OBTAINED AFTER 10 MC RUNS BY DIFFERENT METHODS WITH SIMULATED DATA.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>SSWNMF</th>
<th>SGSNMF</th>
<th>TV-RSNMF</th>
<th>RSNMF</th>
<th>GLNMF</th>
<th>$L_{1/2}$-NMF</th>
<th>VCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR=20dB</td>
<td>0.0636 ± 0.40%</td>
<td>0.0782 ± 0.50%</td>
<td>0.0679 ± 0.30%</td>
<td>0.0731 ± 0.50%</td>
<td>0.0724 ± 0.50%</td>
<td>0.0744 ± 0.40%</td>
<td>0.1358 ± 0.30%</td>
</tr>
<tr>
<td>SNR=30dB</td>
<td>0.0122 ± 0.01%</td>
<td>0.0176 ± 0.03%</td>
<td>0.0131 ± 0.03%</td>
<td>0.0138 ± 0.05%</td>
<td>0.0144 ± 0.04%</td>
<td>0.0142 ± 0.04%</td>
<td>0.0350 ± 0.06%</td>
</tr>
<tr>
<td>SNR=40dB</td>
<td>0.0029 ± 0.02%</td>
<td>0.0033 ± 0.03%</td>
<td>0.0036 ± 0.02%</td>
<td>0.0041 ± 0.04%</td>
<td>0.0044 ± 0.05%</td>
<td>0.0037 ± 0.04%</td>
<td>0.0125 ± 0.05%</td>
</tr>
<tr>
<td>SNR=50dB</td>
<td>0.0012 ± 0.02%</td>
<td>0.0019 ± 0.02%</td>
<td>0.0014 ± 0.03%</td>
<td>0.0020 ± 0.04%</td>
<td>0.0023 ± 0.04%</td>
<td>0.0024 ± 0.03%</td>
<td>0.0049 ± 0.06%</td>
</tr>
</tbody>
</table>

TABLE II
AVERAGE RMSE SCORES (ALONG WITH THEIR STANDARD DEVIATIONS) OBTAINED AFTER 10 MC RUNS BY DIFFERENT METHODS WITH SIMULATED DATA.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>SSWNMF</th>
<th>SGSNMF</th>
<th>TV-RSNMF</th>
<th>RSNMF</th>
<th>GLNMF</th>
<th>$L_{1/2}$-NMF</th>
<th>VCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR=20dB</td>
<td>0.1339 ± 0.20%</td>
<td><strong>0.1322 ± 0.40%</strong></td>
<td>0.1342 ± 0.30%</td>
<td>0.1426 ± 0.40%</td>
<td>0.1434 ± 0.60%</td>
<td>0.1430 ± 0.50%</td>
<td>0.1704 ± 0.30%</td>
</tr>
<tr>
<td>SNR=30dB</td>
<td><strong>0.0386 ± 0.20%</strong></td>
<td>0.0391 ± 0.30%</td>
<td>0.0420 ± 0.20%</td>
<td>0.0426 ± 0.30%</td>
<td>0.0429 ± 0.30%</td>
<td>0.0432 ± 0.20%</td>
<td>0.0548 ± 0.20%</td>
</tr>
<tr>
<td>SNR=40dB</td>
<td><strong>0.0122 ± 0.03%</strong></td>
<td>0.0148 ± 0.05%</td>
<td>0.0143 ± 0.04%</td>
<td>0.0147 ± 0.05%</td>
<td>0.0150 ± 0.04%</td>
<td>0.0153 ± 0.03%</td>
<td>0.0164 ± 0.10%</td>
</tr>
<tr>
<td>SNR=50dB</td>
<td><strong>0.0041 ± 0.02%</strong></td>
<td>0.0059 ± 0.05%</td>
<td>0.0050 ± 0.03%</td>
<td>0.0055 ± 0.03%</td>
<td>0.0064 ± 0.04%</td>
<td>0.0061 ± 0.04%</td>
<td>0.0087 ± 0.08%</td>
</tr>
</tbody>
</table>

unmixing calculations. Fig. 1 (b)-(j) show the abundance maps of the endmembers. In the wake of creating the datacube, the scene was tainted with i.i.d. Gaussian noise, with four degrees of the signal-to-noise ratio (SNR): 20, 30, 40 and 50 dB.
B. Experimental Settings

For the proposed SSWNMF technique, there is just a single regularization parameter $\lambda$ that should be tuned heretofore for various datasets, while different parameters can be viewed as hyperparameters and given explicit values. To take into account a reasonable examination, in all the reproduced tests the underlying endmember matrix is fixed (acquired by the VCA [4]), and the underlying abundance maps were produced by FCLS [38]. Subsequently, the initial endmember $M$ and abundance $S$ matrices both fulfill the nonnegative constraint. It ought to be noticed that the parameters associated with the contender’s strategies are set by the directions and demos gave by the creators in the original contributions. We emphasize that all the algorithms were implemented using MATLAB R2016a and tested in the same computing environment: a desktop computer equipped with an Intel Core 7 Duo central processing unit (at 3.6 GHz) and 16 GB of RAM memory.

C. Performance Comparisons

In this test, simulated data tainted by various SNRs are utilized to assess the power of the proposed SSWNMF unmixing technique. Four distinctive SNR levels are thought of, with a stage size of 10 dB and a range from 20 dB to 50 dB. Table I shows the SAD outcomes got by various tried strategies on the synthetic data under various SNR levels, and Table II likewise displays the RMSE results. It should be noted that all the results are averaged after 10 independent Monte Carlo runs. It can be seen from Table I that the proposed SSWNMF method obtains lower SAD scores than SGSNMF, TV-RSNMF, RSNMF, GLNMF, $L_{1/2}$-NMF and VCA. This reveals the benefits of including spectral and spatial weighting factors in the sparse NMF model. From Table II, we can see that the SSWNMF obtains lower or similar RMSE results than the other tested methods in most cases. Moreover, SSWNMF, SGSNMF and TV-RSNMF provide lower RMSE values than RSNMF, GLNMF, $L_{1/2}$-NMF and VCA, which demonstrates the advantage of pursuing spatial consistency in the obtained results. Likewise, our SSWNMF accomplished better or similar results than TV-RSNMF, which demonstrates that the consideration of a spatial weighting factor in the sparse NMF model can additionally incorporate a piecewise smooth structure in the resulting abundance maps. In summary, it very well may be inferred that our spectral and spatial weighted technique gives the possibility to improve NMF unmixing execution in this situation.

So as to additionally exhibit the presentation of various calculations, Fig. 2 shows the abundance maps estimated for endmember 1 when the SNR is 30dB. It ought to be noticed that just the abundance map of endmember 1 was chosen, since all endmembers provided abundance maps with comparative conduct. Fig. 2 additionally presents the difference maps between the evaluated abundances and the original ones. It very well may be unmistakably seen that the abundance estimation results got by VCA-FCLS look wrong. Likewise, from those difference maps, we can see that the outcomes got by SSWNMF are close to the ground-truth ones, which display more clear spatial structure and surface subtleties. Subsequently, compared with $L_{1/2}$-NMF and RSNMF calculations, it gives better abundance estimation results. At last, the abundance estimation results acquired by SSWNMF are slightly better than those gave by TV-RSNMF calculation. This further demonstrates that the consideration of spatial weighting factor in the sparse NMF model can advance the spatial connection of the arrangement and improve the exactness of unmixing.

Regarding the convergence of Algorithm 1, it stops
TABLE III
Computational times (seconds), SADs and AREs along with their standard deviations obtained after 10 MC runs by different tested methods for the Urban data set

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>SSWNMF</th>
<th>SGSNMF</th>
<th>TV-RSNMF</th>
<th>RSNMF</th>
<th>GLNMF</th>
<th>$L_{1/2}$-NMF</th>
<th>VCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asphalt Road</td>
<td>0.0782 ± 3.29%</td>
<td>0.0804 ± 4.01%</td>
<td><strong>0.0770 ± 2.97%</strong></td>
<td>0.0869 ± 3.81%</td>
<td>0.1008 ± 3.19%</td>
<td>0.0889 ± 2.88%</td>
<td>0.2246 ± 3.44%</td>
</tr>
<tr>
<td>Grass</td>
<td>0.1490 ± 3.58%</td>
<td>0.1516 ± 3.25%</td>
<td>0.1495 ± 3.54%</td>
<td>0.1594 ± 3.62%</td>
<td>0.1531 ± 3.06%</td>
<td><strong>0.1452 ± 3.57%</strong></td>
<td>0.1981 ± 3.39%</td>
</tr>
<tr>
<td>Tree</td>
<td><strong>0.1173 ± 3.46%</strong></td>
<td>0.1199 ± 3.36%</td>
<td>0.1269 ± 4.02%</td>
<td>0.1457 ± 4.29%</td>
<td>0.1424 ± 3.79%</td>
<td>0.1509 ± 3.18%</td>
<td>0.2137 ± 2.41%</td>
</tr>
<tr>
<td>Roof</td>
<td><strong>0.0713 ± 3.61%</strong></td>
<td>0.0731 ± 3.54%</td>
<td>0.0746 ± 4.09%</td>
<td>0.0849 ± 3.90%</td>
<td>0.0986 ± 4.62%</td>
<td>0.0863 ± 4.06%</td>
<td>0.2673 ± 3.77%</td>
</tr>
<tr>
<td>Metal</td>
<td><strong>0.1241 ± 2.76%</strong></td>
<td>0.1250 ± 3.81%</td>
<td>0.1247 ± 3.53%</td>
<td>0.1324 ± 4.15%</td>
<td>0.1370 ± 4.28%</td>
<td>0.1334 ± 3.90%</td>
<td>0.1848 ± 3.68%</td>
</tr>
<tr>
<td>Dirt</td>
<td>0.0802 ± 3.17%</td>
<td>0.0859 ± 3.91%</td>
<td>0.0849 ± 3.92%</td>
<td><strong>0.0798 ± 3.77%</strong></td>
<td>0.1059 ± 3.96%</td>
<td>0.1063 ± 3.54%</td>
<td>0.1992 ± 3.43%</td>
</tr>
<tr>
<td>Mean SAD</td>
<td>0.1034</td>
<td>0.1060</td>
<td>0.1063</td>
<td>0.1149</td>
<td>0.1230</td>
<td>0.1185</td>
<td>0.2146</td>
</tr>
<tr>
<td>ARE</td>
<td>0.0048 ± 0.72%</td>
<td>0.0061 ± 0.67%</td>
<td>0.0055 ± 0.81%</td>
<td>0.0053 ± 0.98%</td>
<td>0.0069 ± 0.85%</td>
<td><strong>0.0044 ± 0.76%</strong></td>
<td>0.0119 ± 0.66%</td>
</tr>
<tr>
<td>Time (Seconds)</td>
<td>141.79</td>
<td>175.17</td>
<td>189.06</td>
<td>68.43</td>
<td>590.71</td>
<td>90.55</td>
<td>8.07</td>
</tr>
</tbody>
</table>

Fig. 3. The residual: $\max(||M^{(t+1)} - M^{(t)}||_F, ||S^{(t+1)} - S^{(t)}||_F)$ as a function of the number of iterations for the complete algorithm.

when the maximum iteration number is reached or when $\max(||M^{(t+1)} - M^{(t)}||_F, ||S^{(t+1)} - S^{(t)}||_F)$ ≤ threshold (a condition that we have empirically found to be quite common in practice). For illustrative purposes, Fig. 3 shows the convergence curve obtained for our SSWNMF for the simulated datasets with SNR=30dB. The number of iterations is set to 1000. It can be seen that the proposed SSWNMF algorithm exhibits good convergence.

V. EXPERIMENTS WITH REAL DATA

In this section, two benchmark hyperspectral datasets are adopted for surveying the unmixing execution of our recently proposed SSWNMF technique. The first dataset is a Urban scene procured by the Hyperspectral Digital Imagery Collection Experiment (HYDICE), and the second dataset is the notable Cuprite scene, which was obtained by the Airborne Visible Infrared Imaging Spectrometer (AVIRIS). The average reconstruction error [39], [40] ARE = $\sqrt{\frac{1}{MN} \sum_{k=1}^{d} \sum_{j=1}^{N} ||\hat{Y}_{k,j} - Y_{k,j}||^2}$ is used for quantitative evaluation.

A. Experiments on the HYDICE Urban Dataset

The Urban dataset has a spatial size of $307 \times 307$ pixels and 210 spectral bands. Because of thick water vapor and environmental impacts, the channels: 1-4, 76, 87, 101-111, 136-153 and 198-210 are removed and we hold 162 channels for tests. The spatial resolution of this scene is 2×2 m², and the spectral resolution is 10 nm, covering the frequency from 0.4 to 2.5 μm. The false color composition and the six comparing endmembers are shown in Fig. 4: Asphalt Road, Grass, Tree, Roof, Metal and Dirt. The reference signatures were gathered from the spectral library downloadable from².

Table III shows the SAD and ARE scores obtained by the six considered unmixing methods for the Urban scene, including the SAD scores for each ground class and their average value. We can see from Table III that the proposed SSWNMF method achieves very good performance in terms of SAD, and can well

²http://www.tec.army.mil/Hypercube
recognize most endmembers. This shows the importance of the spectral-spatial weighted procedure to improve NMF-based unmixing. Besides, when compared with GLNMF, $L_{1/2}$-NMF and VCA, the SSWNMF, SGSNMF, TV-RSNMF and RSNMF methods achieve lower mean SAD scores, which demonstrates that the weighted sparse regularizer offers significant advantages in terms of NMF-based unmixing. Furthermore, the results obtained by SSWNMF and SGSNMF are better than those provided by TV-RSNMF, which demonstrates that the incorporation of spatial weights for spatial characterization in the sparse NMF model is compelling. In addition, the results obtained by SSWNMF are slightly better than those provided by SGSNMF. This indicates that the combination of spectral and spatial weighting factors can play a significant role in the task of improving unmixing performance. For illustrative purposes, Fig. 5 presents a correlation of the endmember signatures obtained by SSWNMF and the reference signatures got from the spectral library. It very well may be seen that the endmember signatures are all around correlated in spectral terms with respect to the reference partners. Additionally, Fig. 6 shows the abundance maps evaluated by our proposed SSWNMF calculation, where the grayscale esteem (from dark to white) speak to abundance esteem (from 0 to 1). It can likewise be seen that the assessed abundance maps show extremely clear spatial and surface subtleties for each ground class. Generally speaking, the trial results got for the Urban data demonstrate that our spectral-spatial weighted system brings advantage for unmixing purposes.

B. Experiments on the AVIRIS Cuprite Dataset

The second hyperspectral scene utilized in this tests is the notable AVIRIS Cuprite data, which is accessible online in
Fig. 7. USGS map displaying the location of different minerals in the Cuprite mining district in Nevada.

reflectance units. The spatial size of the Cuprite image is $250 \times 191$, with 224 spectral bands in the range of 0.4-2.5 $\mu$m and spatial resolution of 10 mm. We removed bands 1-6, 105-115, 150-170, and 223-224 because of water absorption and low SNR, we keep an aggregate of 183 bands for test. The Cuprite site is broadly known in mineralogically terms and there are a few uncovered minerals, all recorded in the USGS library (utilized in this paper to acquire reference signatures). For illustrative purposes, Fig. 7 shows a mineral map delivered by USGS in 1995, which is accessible online. Regarding the parameters involved in each unmixing algorithm, we follow the settings in the simulated experiments.

It ought to be referenced that, because of the unpredictable mineral composition in this scene, it is hard to decide the number of endmembers $p$. As per the examination in [41], we exactly set $p = 11$ for the thought about scene. Moreover, visual interpretation is likewise utilized in this investigation to check the estimation of $p$ to affirm that the previously mentioned esteem is a sensible gauge of the quantity of endmembers.

For the initialization of the SSWNMF streamlining process, $M$ is introduced by utilizing the equivalent endmember extraction strategy as in [42]. In the first place, we run the VCA strategy $t$ times and hold the simplex of greatest volume. For VCA, this bodes well given the way that the calculation utilizes random directions to discover the limits of the simplex. It should be underlined that, for $t = 30$, this strategy just takes 2 seconds on a standard PC. Next, the dataset is projected to an inflated simplex acquired by permitting the abundances to take negative values. We at that point apply all the contending

### Table IV

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>SSWNMF</th>
<th>SGNSMF</th>
<th>TV-RSNMF</th>
<th>RSNMF</th>
<th>GLNMF</th>
<th>$L_{1/2}$-NMF</th>
<th>VCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alunite</td>
<td>$0.1497 \pm 0.397%$</td>
<td>$0.1238 \pm 4.01%$</td>
<td>$0.1204 \pm 4.37%$</td>
<td>$0.1189 \pm 4.39%$</td>
<td>$0.1353 \pm 3.83%$</td>
<td>$0.1496 \pm 3.32%$</td>
<td>$0.1574 \pm 3.71%$</td>
</tr>
<tr>
<td>Buddingtonite</td>
<td>$0.0958 \pm 4.69%$</td>
<td>$0.1021 \pm 3.47%$</td>
<td>$0.0903 \pm 5.08%$</td>
<td>$0.1342 \pm 4.72%$</td>
<td>$0.1437 \pm 3.62%$</td>
<td>$0.1441 \pm 4.16%$</td>
<td>$0.1412 \pm 3.74%$</td>
</tr>
<tr>
<td>Chalcedony</td>
<td>$0.1496 \pm 4.12%$</td>
<td>$0.1221 \pm 4.02%$</td>
<td>$0.1387 \pm 4.01%$</td>
<td>$0.1224 \pm 4.19%$</td>
<td>$0.1341 \pm 2.98%$</td>
<td>$0.1520 \pm 3.43%$</td>
<td>$0.1514 \pm 3.83%$</td>
</tr>
<tr>
<td>Kaolin 1</td>
<td>$0.0885 \pm 2.94%$</td>
<td>$0.0986 \pm 3.18%$</td>
<td>$0.1097 \pm 3.47%$</td>
<td>$0.0955 \pm 3.07%$</td>
<td>$0.0967 \pm 4.01%$</td>
<td>$0.0825 \pm 4.66%$</td>
<td>$0.0736 \pm 4.42%$</td>
</tr>
<tr>
<td>Kaolin 2</td>
<td>$0.1206 \pm 3.67%$</td>
<td>$0.1375 \pm 3.48%$</td>
<td>$0.1213 \pm 3.82%$</td>
<td>$0.1396 \pm 4.11%$</td>
<td>$0.1356 \pm 3.91%$</td>
<td>$0.1402 \pm 4.18%$</td>
<td>$0.1420 \pm 4.16%$</td>
</tr>
<tr>
<td>Montmorillonite</td>
<td>$0.0651 \pm 3.08%$</td>
<td>$0.0705 \pm 3.36%$</td>
<td>$0.0783 \pm 3.95%$</td>
<td>$0.0744 \pm 3.12%$</td>
<td>$0.0838 \pm 4.28%$</td>
<td>$0.0876 \pm 2.91%$</td>
<td>$0.0974 \pm 3.39%$</td>
</tr>
<tr>
<td>Muscovite</td>
<td>$0.1024 \pm 4.24%$</td>
<td>$0.1061 \pm 3.18%$</td>
<td>$0.1131 \pm 2.88%$</td>
<td>$0.0997 \pm 3.46%$</td>
<td>$0.0961 \pm 3.77%$</td>
<td>$0.0889 \pm 3.03%$</td>
<td>$0.1007 \pm 3.31%$</td>
</tr>
<tr>
<td>Nontronite</td>
<td>$0.1138 \pm 4.15%$</td>
<td>$0.1046 \pm 3.80%$</td>
<td>$0.0911 \pm 3.49%$</td>
<td>$0.0832 \pm 4.18%$</td>
<td>$0.0953 \pm 3.42%$</td>
<td>$0.1038 \pm 4.46%$</td>
<td>$0.0772 \pm 2.10%$</td>
</tr>
<tr>
<td>Sphene</td>
<td>$0.1024 \pm 3.79%$</td>
<td>$0.1179 \pm 4.02%$</td>
<td>$0.1190 \pm 2.97%$</td>
<td>$0.1134 \pm 2.54%$</td>
<td>$0.1291 \pm 4.21%$</td>
<td>$0.1252 \pm 5.18%$</td>
<td>$0.1277 \pm 4.08%$</td>
</tr>
<tr>
<td>Pyrope</td>
<td>$0.1106 \pm 3.32%$</td>
<td>$0.1208 \pm 3.83%$</td>
<td>$0.1253 \pm 3.10%$</td>
<td>$0.1469 \pm 3.12%$</td>
<td>$0.1318 \pm 3.18%$</td>
<td>$0.1123 \pm 4.91%$</td>
<td>$0.1437 \pm 3.76%$</td>
</tr>
<tr>
<td>Jarosite</td>
<td>$0.1424 \pm 4.90%$</td>
<td>$0.1426 \pm 3.72%$</td>
<td>$0.1395 \pm 4.18%$</td>
<td>$0.1471 \pm 4.41%$</td>
<td>$0.1450 \pm 3.56%$</td>
<td>$0.1423 \pm 3.17%$</td>
<td>$0.1464 \pm 3.35%$</td>
</tr>
<tr>
<td>Mean SAD</td>
<td>$0.1128$</td>
<td>$0.1133$</td>
<td>$0.1133$</td>
<td>$0.1159$</td>
<td>$0.1206$</td>
<td>$0.1208$</td>
<td>$0.1235$</td>
</tr>
<tr>
<td>ARE</td>
<td>$0.0043 \pm 0.05%$</td>
<td>$0.0055 \pm 0.06%$</td>
<td>$0.0050 \pm 0.04%$</td>
<td>$0.0057 \pm 0.07%$</td>
<td>$0.0040 \pm 0.06%$</td>
<td>$0.0048 \pm 0.07%$</td>
<td>$0.0113 \pm 0.08%$</td>
</tr>
<tr>
<td>Time (Seconds)</td>
<td>149.15</td>
<td>179.33</td>
<td>201.75</td>
<td>45.85</td>
<td>240.46</td>
<td>71.51</td>
<td>12.01</td>
</tr>
</tbody>
</table>
unmixing calculations to the regularized data $Y_{\text{reg}} = \hat{M}_{\text{vca}} \hat{S}_t$, where $\hat{M}_{\text{vca}}$ is the mixing matrix acquired by VCA and $\hat{S}_t$ is the outcome assessed by FCLS. It ought to be noticed that this strategy is compelling and can significantly improve unmixing calculations.

Table IV presents the SAD and ARE scores for SSWNMF, SGSNMF, TV-RSNMF, RSNMF, GLNMF, $L_{1/2}$-NMF and VCA, separately. It can be seen from Table IV that the proposed SSWNMF obtained the best mean SAD scores. In terms of individual scores, those obtained by SSWNMF are
Fig. 8 shows the abundance maps assessed by the proposed SSWNMF calculation. The estimation results have a good relationship with the geological maps of the Cuprite dataset in Fig. 7, and some endmembers can be very much distinguished (for example, Alunite, Buddingtonite and Montmorillonite). Likewise, Fig. 9 shows the examination between the endmembers extracted by our technique and their relating USGS library signatures. It ought to be noticed that, because of the multifaceted nature and variability of some mineral signatures, we contrasted the evaluated endmembers and the USGS library signatures. We can see from the Fig. 9 that the endmembers obtained by SSWNMF furnish a generally excellent match with respect to the comparing USGS library signatures. In outline, the exploratory outcomes uncover that the proposed SSWNMF can deliver extremely competitive outcomes contrasted with those gave by different calculations, for example, TV-RSNMF, RSNMF, GLNMF, $L_{1/2}$-NMF and VCA.

Last but not least, we emphasize that the computational times of all considered algorithms are reported on Tables III and IV. It can be observed from these tables that, among the methods with spatial information, the proposed SSWNMF is faster than SGSNMF, TV-RSNMF and GLNMF.

VI. CONCLUSIONS AND FUTURE WORK

In this work, a new spectral-spatial weighted sparse NMF model has been created for blind unmixing of hyperspectral data. Our recently proposed SSWNMF calculation includes a spatial weighting factor that is utilized to promote the piecewise smoothness in the abundance maps, while the spectral weighting factor is utilized to upgrade the sparsity of the arrangement. The proposed SSWNMF model can be solved by a multiplicative iterative technique. Our test results clearly reveal that the proposed SSWNMF is able to obtain steady and precise unmixing results. In addition, our new strategy exhibits significantly less computational complexity than other methods. In future work, we will explore deep autoencoder systems [43], [44] and dictionary learning [45] for hyperspectral unmixing.

REFERENCES


